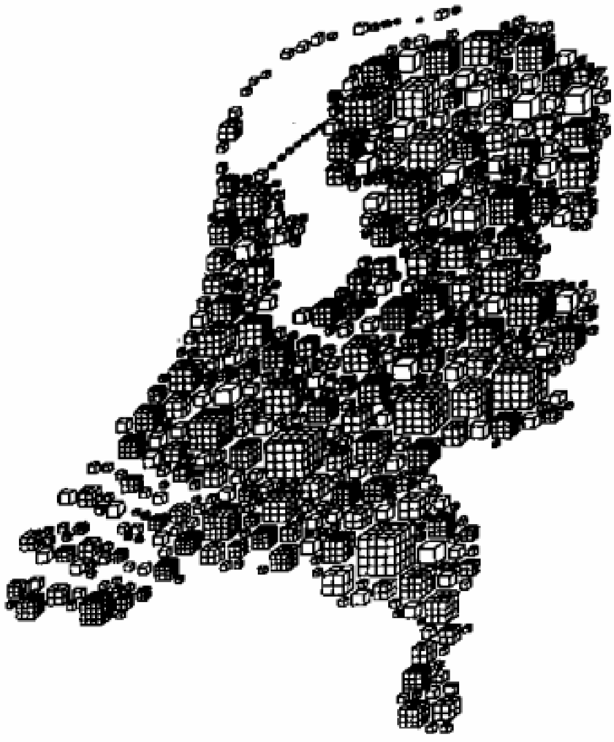
CFF

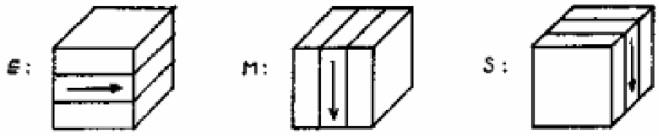
CUBISM FOR FUN



Notation of slice and cube moves

In our articles we often use slice and cube moves. There are many ways to represent these moves, and we use the slice notation as first used by Frans Schiereck:

Introduce the moves E(quator), M(iddle) and S(tanding), being the following central layer moves respectively:



Now, how to count processes involving these moves? Singmaster's notation does not include central layer moves; instead he moves the outer faces. Our extended notation is Singmaster's with the moves M, E and S added. There are many instances where clarity is lost if Singmaster's notation is used: In working with larger cubes it is impossible to stick to it, and in the desciption of Marc Waterman's algorithm it would be extremely annoying and illegible to refrain from using M.

Whatever notation is used, a move will be the rotation of one layer over 90, -90 or 180 degrees. Thus M is one move and RL' is two moves, so a process containing c central layer moves is c moves longer in Singmaster's notation than in the extended notation.

To describe a move of the cube as a whole, we use C_R , C_F etc., e.g. C_R = RM'L'. These cube moves all have length 0.

Marc Waterman and Anneke Treep

A FAST METHOD FOR THE FIRST FACE; AS USED BY MARC WATERMAN

By Marc Waterman and Anneke Treep

In cube literature very little attention has been paid to the way one face is solved. The few methods described are rather slow. Though it is impossible to describe a complete method that is easy to understand and still three times faster than any book method, it is very well possible to give a general outline of the way Marc solves the first face, which he does in an average time of 6 to 7 seconds.

His method can roughly be divided in the following steps:

1. Two adjacent corners;
2. The other two corners;
3. The four edges and the centre.

Most people find it convenient to start with the same colour every time they solve the cube. The first face will be formed on the U side of the cube.

Notation.

In order to show the different corner configurations we will use the diagrams Minh Thai has used in his book 'The winning Solution'. A diagram shows the cube in a 'flattened' way (fig. 1) with the U face in the middle and the side faces around it.

The cubies are represented by circles: a complete cube is shown in fig. 2. The D centre is the only cubie that cannot be represented in such a diagram.

In a circle letters can be placed to indicate the colours of the facelets. In this article we will show the facelets of U colour in black, and only relevant cubies will be shown.

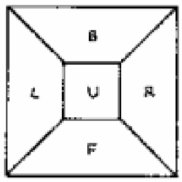


fig. 1.

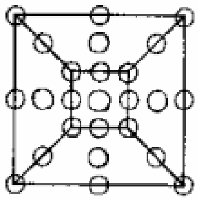


fig. 2.

The method, step by step.

1. Put two adjacent U corners in the correct way next to each other. Do not worry about the centres yet! This can always be done in at most three moves, usually it takes only two. Now hold the cube in such a way that the U facelets of the corners are in the U face, as in fig 3.

2. It is possible to solve the other two U corners one by one, but this often takes too many moves. It is better to solve them together. Alas, it would take too much room to give solutions to all possible cases, so we give only the very hard ones.

Most if not all cases can be solved in at most seven moves, mainly with the use of processes in the <R,D> group (i.e. processes with only R and D moves). Here are the most difficult cases:

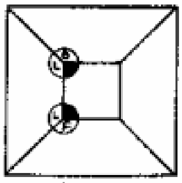
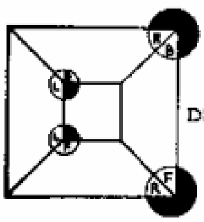
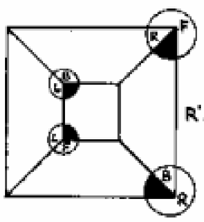


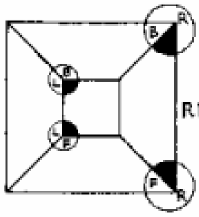
fig. 3.



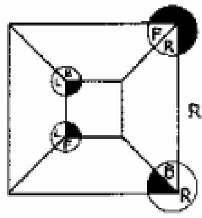
$DRD^{2}RD^{3}R$



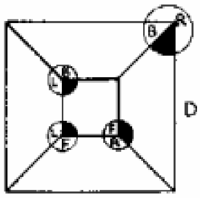
R'D'RD'R



RDR²D'R



RFDF²R'F



DFDF'R²

3. Now the four edges and the centre still have to be solved. Here too it would take too much space to show all possible situations, so we will give a general outline of the method and some very short processes that you may apply invertedly to see what types of processes are used.

Having solved the U corners, there often are one or more U edges in the D face. Then one can be solved with a middle layer move after it has been brought to the correct slice. Sometimes a second edge can be solved in the same way, provided it has its home in a middle layer that does not contain another solved U edge. In such a case it is very often possible to solve the U centre together with the second U edge:

- If both the U centre and the U facelet of the edge are in the D face this is very simple.
- If both the U centre and the U facelet of the edge are in the same side face (e.g. at R and RD) it is also very simple.
- If the U centre is in a side face and the edge is in the D face with the U facelet in a side face, put them together with a D move and you have one of the easier cases.

In other cases it is more difficult and the U centre may be left out.

To solve the other U edges the following well known processes may be used:
F'EF , F'E²F , F'E'F , MDM' , MD²M' .

If the centre has not been corrected together with the second edge, it can be solved with one of the other edges. Here are some processes to do this:

B'MB	BM²B'
B'M'B or MD'E'M'	B'E'D'MEDB
MD²E²M'	B'E²D²ME²D²B
MD²E'M'	B'M²B²MB'
MD'E²M'	B'M²BFE'F'

To find out what the effect of one of these processes is, turn the inversion of a process to achieve the situation it solves.

<u>Some more tips</u>

- At contests it is usually allowed to watch the cube some 15 seconds before time starts. In these 15 seconds you may pay attention to two things:
  1) Decide which two corners to solve first. Only seldom is it possible to see how the other two corners must then be solved.
  2) Often you can use a slice move to solve an edge that belongs next to one or both of the first two corners before starting on the other two corners. The edge will not be affected when solving the last two corners.
     It is not recommended to solve a second edge before all corners are solved, as it may then take more moves to solve the centre.
- If the centre is solved together with the second edge, the last two edges can often be solved together: The process that solves the third edge ends with the inverse of the move that the process for the fourth edge starts with, so you may save two moves. This means that is even saves a move to produce such a situation with one extra move! Here are some examples:
  BEL'C_LD²M' , L'E'R$C_{R'}$DM', F²M²F²M² , R_a'E'R_a .

The algoritm of Marc Waterman and Daan Krammer

Introduction

The basic algorithm described in this article has been developed by Marc Waterman and Daan Krammer in 1981. With a lot of practice an average solving time of 25 to 30 seconds should be possible. Marc of course uses a lot of additional processes and shortcuts and his long experience to achieve his average of 18 seconds on his own cube.
The algoritm is not only extremely fast, but also very interesting as it is not just a bunch of tricks to learn by heart: Espescially the edge-solving stages require a deeper understanding of the underlying ideas.

As will be seen, the speed of the system comes from the use of mostly U-, R- and RL-slice moves when solving edges. Thus hardly any time is lost in changing the way the cube is held.

First, let's define some words and notations to be used:

ring : The slice between the R and L faces.
redge : An edge that has its "home" in the R face.
midge : An edge that has its "home" in the ring.
M : A quarter turn of the ring, which moves UF to FD:

In short, the algorithm is as follows:

1) The L face is solved.
2) The R corners are positioned and oriented simultaneously.
3) The redges are solved, and the orientation of all midges is corrected.
4) The midges are positioned.

We are aware of the fact that both the text and the notations in the tables may prove to be difficult to understand. Still, we feel that to grasp the underlying ideas some exploring of your own is necessary anyway. The best way to overcome these problems is to apply the inversion of a process from the table you have difficulties with and compare the situation thus achieved with the notation and the text. It is also very helpful to turn a process very slowly and follow the pieces involved.

Solving the cube

1. The L face

The fastest method is to first put all L corners correct in respect to each other, and then put the edges into place. The centre should be done along with the second edge.
The L face may thus be solved within ten seconds' time.

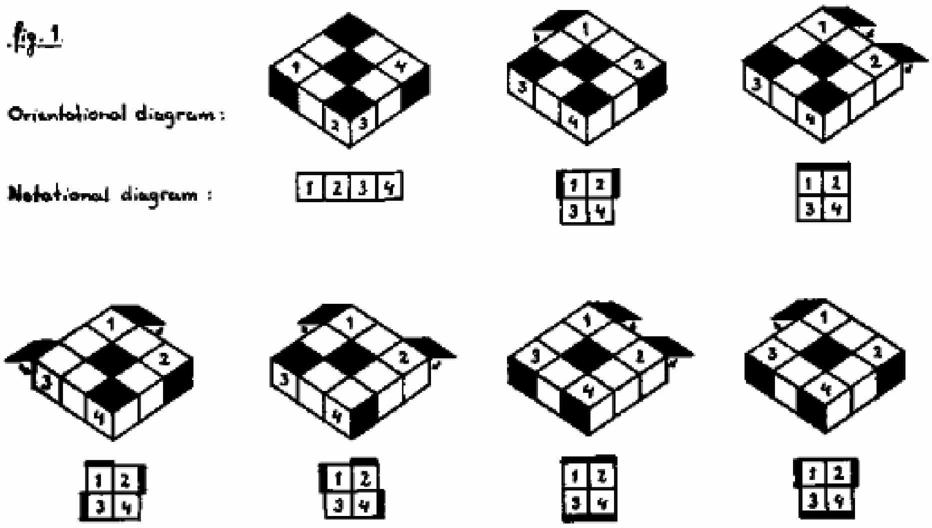
2. The R corners

When solving the R corners, the cube is temporarily held in such a way that the R face becomes U face.
The corners are positioned and oriented simultaneously. There are 43 different situations, and each is recognized by looking at the corners only. Thus no extra U turns are necessary, nor the comparison of the corners with other pieces.

The 43 processes may be found in table 1 (the tables can be found at the end of this article). To determine the process to be used, first compare the orientation of the corners with the diagrams in fig. 1.

The black facelets are of the U colour. In each diagram four other facelets are numbered (as you can see, it depends on the orientation of the corners which four). These are the facelets that are used to recognize the situation once the orientational configuration has been determined.

fig. 1



In the corresponding notational diagram the numbers have yet to be replaced by the letters F, B, L or R. These letters indicate how the colours of the numbered facelets are placed in respect to each other on a clean cube. As a rule, the colour that appears most is referred to as F(ront).

NB. When determining the situation the colour F is not necessarily the colour of the centre of the front face, because we only need the placement of the colours in respect to each other.

3. Redges and orienting of midges

The cube is now held as it originally was, so the only pieces that may (and most probably do) need solving are the edges or the R face and those of the ring.

In three out of four cases no redge is completely solved yet, and one, two or three redges are in the ring.
In these situations first two redges are solved, while keeping at least one redge that already was in the R face in this face, though it may stay wrongly positioned and/or oriented. Then the remaining redges are solved and the orientation of the midges is corrected simultaneously.

In the other 25% of the cases the solution is a little different, though based on the same principles.

In this section we will now deal with the following subjects:
- Agreements on the notation
- A description of the basic method
- Methods for good and bad luck situations
- Complications

The tables referred to in these sections are to be found at the end of the article.

Agreements on the notation

To describe the situations that arise a wholly new notation has been developed. Four aspects of this notation need a clear introduction. These concern:

- The redges in the ring
- The redges in the R face
- The move (R) in a process
- Midges in the ring

Most of the tables use at least one of these notations or a slight variation. The reader should therefore examine this section carefully.

The redges in the ring

In order to be able to apply a process the pieces involved should of course be in the correct position. The required position for a redge in the ring is denoted by two letters, the first of which indicating the face that is to contain the facelet of the R colour. The desired position of a redge can be obtained by a ring move, i.e. M, M' or M^2.

It will be clear that, as every situation has a mirror image, most processes have two entries as only one can be achieved by using ring moves.

In table no. 2 there are two redges in the ring. By using ring moves one of the situations given can be achieved, keeping in mind that the first letter of both positions indicates the facelet of the R colour.

The redges in the R face

The R face contains four edge positions that may or may not contain a redge. An edge position that is not solved, i.e. filled properly with the right redge, is called a hole.
There are three types of holes, each denoted by a square:

□ : A hole that contains an unflipped redge.
(The R facelet of the redge is in the R face)

▨ : A hole that contains a flipped redge.
(The R facelet of the redge is not in the R face)

⬚ : A hole filled with a midge, commonly called an empty hole.

An arrow from one square to another indicates that the redge should be moved from one hole to the other to be solved during the process.
A double arrow indicates an exchange of the redges.

An example: ▨→⬚ means:

A redge is positioned in the R face with its R facelet not in the R face. In order to be solved it should be moved to an other hole that is currently filled with a midge.

At the beginning of a process one of the holes that will be affected must be at the RU position. This hole can be either filled or empty, which is indicated by a small circle:

○ : The hole at RU is empty.

● : The hole at RU is not empty.

The desired situation can be achieved by means of a move of the R face.

The move (R) in a process

In many cases in the course of a process a different hole is moved to the RU position by a move of the R face. As the relative position of the holes in the R face is irrelevant to the structure of the process, the required move may be R, R' or R^2. It is therefore noted as (R).
Sometimes also (R)' occurs, which means that the move executed at (R) earlier in the process should be inverted.

Midges in the ring

In tables 4 and 5 one redge and three midges are in the ring. Each midge can be flipped or not flipped. Not flipped means that a facelet of the midge has the same colour as the centre it is adjacent to, or the colour of the opposite centre.
To show wheter a midge is flipped or not a X or 0 is used:

X : midge is flipped

0 : midge is not flipped

The orientations of the midges are shown in a series of three X or O's. The first X or O represents the midge that is positioned in the same face as the R facelet of the redge. The second represents the midge diagonally opposite the redge, and the last X or O represents the last midge. One might thus say that one starts at the midge that the R facelet "points to", and then follow the midges round the ring.

In tables 3 and 5a all midges are in the ring and the orientation of each is given in the corresponding column.

finally we will give a general example of the notation discussed above:

 X 0 X ● DF means:
DB

redge in the R face is positioned in the wrong hole and it is not flipped. It should be moved to a hole that is now empty. The other redge involved (which has to be moved to the hole that is now filled by the first redge) is in the ring. Starting from the R facelet of the redge in the ring we see that the first midge is flipped, the second is not flipped, and the last midge is also flipped. (Shown by XOX)
The filled hole must be held at the RU position (●). Finally, the R facelet of the redge in the ring must be in the D face, the two possibilities DF and DB being mirror images. Both processes are given.

Description of the basic method

Now that you are familiar with the notation used we will describe the basic method for solving the redges and orienting the midges.

The basic method can be applied if one, two or three redges are in the ring, and all are wrongly positioned and/or oriented. The only exceptions are situations where one redge is in the ring and the other three form a 3-cycle in the R face. In such a case see 'complications'.
For those cases where no or all redges are in the ring a method based on the basic algorithm will be given in the next paragraph.

The basic method consists of two steps:

1. Solving two redges

This step solves two redges from the ring or one from the ring and one from the R face. In any case one must make sure that at least one redge that already was in the R face stays there.
It is necessary to do so in order to be able to perform the second step. Though it may at first be difficult to choose the right edges, you will soon get more experienced.
So, you will have to look for two redges that may be solved while leaving a third in the R face. To find such a combination choose a redge in the R face and find two other redges (of which one may be in the R face) that do not have to fill the hole the first chosen redge is in, and that do not come from the hole where the first redge has its home.
For this step tables 2a and 2b are used.

2. Solving the remaining two redges and orienting all midges

If the first step is executed properly, at least one of the two remaining redges is in the R face. Then we can apply a process that solves both redges and orients the midges at the same time:
If both redges are in the R face use table 3.
If one is in the R face and the other is in the ring use table 4.

Methods for good and bad luck situations

A Good luck

1)- In 20,5 % of the possible edge positions/orientations one redge is already solved when the R corners are ready. The other three should then be solved as follows:
1st step Solve two redges using table 2a or 2b, or 3.
2nd step Solve the last redge an orient the midges using table 5.

2)- In 2,2 % of all cases only two redges need solving.
If one redge is already in the R face use table 4.
If both redges are in the R face use table 3.
If both are in the ring solve them as follows:
1st step Solve one of them using the first process of table 5b.
2nd step Solve the other redge and orient the midges using table 5.

3)- In 0,11 % of all cases only one redge needs solving. This is done together with orienting the midges, again using table 5.

4)- Very rarely the R face is complete as the corners are solved. To orient the midges use table 6.

B Bad luck

1)- All four redges are in the ring. This happens with 1,4 % chance. Solve two redges using table 2a, then go to good luck case 2. In fact, a non-standard method exists. We may deal with this in a future issue. If you feel you understand the basic method fully, you may try to figure this out...

2)- In 1,4 % of all cases all four redges are in the R face, but none of them in the right way.
If one of the following situations arises use table 3 to solve two redges:
- all redges are flipped.
- two redges are exchanged and at least two are flipped.
- there are two pairs of exchanged redges, some of them may be flipped.
Then the remaining two can be solved and the midges are oriented again using table 3.

If a three- or even a four-cycle occurs you're in deep trouble... See complications.

Complications

The simplest solution is this: First put one of the redges into the ring using the first process of table 5. This puts you in a situation where the standard method can be used.
There is a more elegant solution, but it is not in the UMR-group, and therefore rather slow. This may also be discussed in future issues.

4. The positioning of the midges

The processes used in this stage are very well known, but still we included them to be complete. (see table no. 7)
Still, when turning the last moves of the third stage one may already insert moves so that this stage may sometimes be avoided. This of course requires a lot of experience.

Conclusions

This system is not only of interest for cube-racers, but some ideas will also prove useful for other systems, such as those that first solve all corners and then all edges.

Marc Waterman and Anneke Treep

An example of the use of the algorithm of Marc Waterman and Dean Krammer

As we presume you are able to complete one face of the cube, we will start our example with one face completed.
To achieve such a situation turn on a clean cube $R^2URU'R^2F'U^2F$.

To solve this following the algorithm, turn the cube so that the F face becomes R face and the U face stays U. Take table 1 and check that situation 6 is involved. Execute the corresponding process.

Now turn the cube so that the U face becomes R face and the F face stays F. Now only the redges and midges need solving.
Check that the only way to solve two redges while keeping a third in the R face is to solve the redges that are in the ring. You can see this as follows: Say you want to solve the redge at RU. Then you are affecting the hole where the redge at RD has its home, so that's no good. If on the other hand you want to solve the redge at RD, it would have to go to the hole at RU, and thus the only other redge in the R face would be affected.
So, the only way out is to solve the redges in the ring. Their corresponding home-holes are at RD and at RB. When solving these redges the redge at RU will therefore stay in the R face.
To solve the two redges we need table 2a, as they are both in the ring. By executing the move M the redges are moved to the required position for process no 5 (check that all other situations cannot be achieved by ring moves). Now we need to move the hole where the redge at FU belongs to the RU position. This is done by the move R'.
Now execute the corresponding process, reading (R) as R', as at this point the hole where the other redge to be solved belongs has to be moved to the RU position. Consequently, at (R)' the move R should be executed.

Of the remaining redges one is in the ring, and one is in the R face, so we now need table 4. The redge in the R face is flipped and it is in the wrong hole, so we need one of the processes from the middle section. To determine which one, we look at the orientation of the midges.
First, find the R facelet of the redge in the ring. It is at the D side of FD, so we start examining the midges at BD. The midge at BD is flipped. Then look at the midge at UB: it is not flipped. The midge at UF is again flipped. This series of orientations therefore is X O X .
So we are at situation 14a or 14b.
The redge in the ring can be moved to BD, but not to FD (remember, the first letter indicates where the R facelet should go). So we need process 14b. Before we can execute it, we move the redge to the correct place by the move M, and as the filled circle indicates that the filled hole should be at RU, we move it there by executing R.
Now apply process 14b, reading R where (R) stands in the process, as at that point the other hole we need has to be brought to RU. Of course, at (R)' you should then apply R'.

Now all redges are solved, and all midges are oriented. To get them to their correct positions apply $U^2D^2MU^2D^2M$.

AN ADVANCED WAY OF SOLVING THE WORST SITUATIONS OF MARC WATERMAN'S ALGORITHM

By Marc Waterman and Anneke Treep

In this article the bad luck situations and complications that arise in the algorithm descibed in issue nr. 14 will be dealt with in an advanced way. That is, with this article all cases of solving redges and orienting midges require two stages.

A. Cases of bad luck

In the case of all redges in the ring, we will now use the following two steps:

1. Solve three redges;
2. Solve the last redge and orient all midges.

In fig. 1 are all essentially different cases with all redges in the ring. The four squares are a representation of the redges as seen 'through' the R face. The thick side of a redge represents the R-facelet of the redge. The letter a, b, c or d represents the other colour of the redge, Note that the situations only differ in the relative orientations of the R-facelets.

There are essentially two classes of redge configurations:

In class I cases it is always possible to find two redges that are in the same face, but that have their R-facelets in opposite faces.

In class II cases no such pair can be found.

Using a ring move (M) one can always achieve one of the following situations:

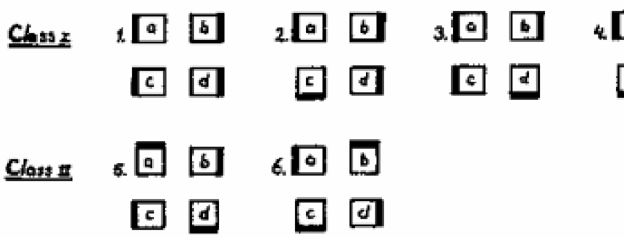


fig.1

The solutions are not given as a set of processes, but as a step by step guide, so that the overall structure becomes clear. The basic idea of the solutions is to solve two redges that are in a favourable position (in class I cases redges a and b, in class II cases redges a and d or b and c after the first U^2 move), and to solve a third redge at a convenient moment.

The expression 'hole x' indicates the edge position where the redge with a facelet of colour x has its home.

Class I

sit. 1-4

1. Turn hole a to RU-position. (R)
2. U^2
3. Turn hole c to RU-position. (R)
4. Sit. 1 or 3: $U'M^2U$
   Sit. 2 or 4: UM^2U'
5. Turn hole b to RU-position. (R)
6. M^2UM^2U'
7. $(R)U^2$

Class II

sit. 5

1. Turn hole a to RU-position. (R)
2. U^2
3. Turn hole c to RU-position. (R)
4. $U'M^2U$
5. Turn hole d to RU-position. (R)
6. $M^2U'M'U$
7. $(R)U^2$

sit. 6

1. Turn hole b to RU-position. (R)
2. U^2
3. Turn hole d to RU-position. (R)
4. UM^2U'
5. Turn hole c to RU-position. (R)
6. M^2UMU'
7. $(R)U^2$

Examining these steps carefully, you will find that the steps 1, 2, 5, 6 and 7 solve two redges. Steps 3 and 4 are inserted to solve a third one. Many of the processes in issue nr. 14 make use of similar ideas.

Having solved three redges, there is only one left. This one is solved, orienting the midges at the same time, using table 5 from issue nr. 14.

An example

Take a clean (i.e. solved) cube, keep your favourite starting colour to the left and turn:

$U^2R'UM^2U'RU^2RM^2U^2RUM^2U'R'U^2$

There are now four redges in the ring. Now find the corresponding situation in fig. 1: it's sit. 1, a class I case. The solution is as follows:

1. R^2, this brings the edge position where the redge at FU belongs to RU.
2. U^2
3. R^2, this brings the edge position where the redge at FD belongs to RU.
4. $U'M^2U$, this puts the redge at FD between its home-corners.
5. R', this brings the edge position where the redge at UF belongs to RU.
6. M^2UM^2U', this puts the redges at FU and UB between their home-corners.
7. $R'U^2$, this restores the L-face.

B. Complications

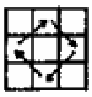
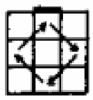
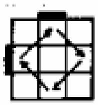
The most loathsome cases are those where three or four redges form a cycle in the R-face. To solve these redges and correct the orientations of the midges, we work through the following two steps:

1. Solve two redges in the R-face that are part of the cycle.
2. Solve the other two redges and orient the midges using tables 3-5 from issue nr. 14.

To solve two (and if you are lucky more) redges from the cycle we use U-processes, so we bring the R-face up by the move $C_{F'}$.

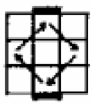
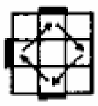
The possible situations are represented by diagrams that show the U-face from above. An arrow shows where the redge at the tail of the arrow has its home. If a redge has a thick side, this means that the redge is flipped.

As will be seen, many different situations get treated with the same process. All 3-cycles and the 4-cycles of the ⟲⟳ -type have mirror images. The processes that solve the mirror images are given after the letters 'Mi'.

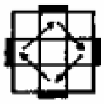
$F^2UM'U^2MUF^2$

Mi: $F^2U'M'U^2MU'F^2$

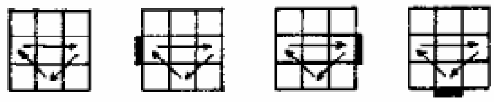
$R'U'RUMU'R'UM'R$

Mi: $RUR'U'M'URU'MR'$



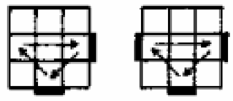
$M'UMU^2M'UM$

Mi: $M'U'MU^2M'U'M$



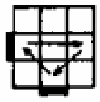
$F^2UM'U^2MUF^2$

<u>Mi</u>: $F^2U'M'U^2MU'F^2$



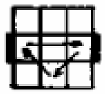
$U^2M'UMU^2M'UM$

<u>Mi</u>: $U^2M'U'MU^2M'U'M$



$U'RM'UR'U'MURU'R'$

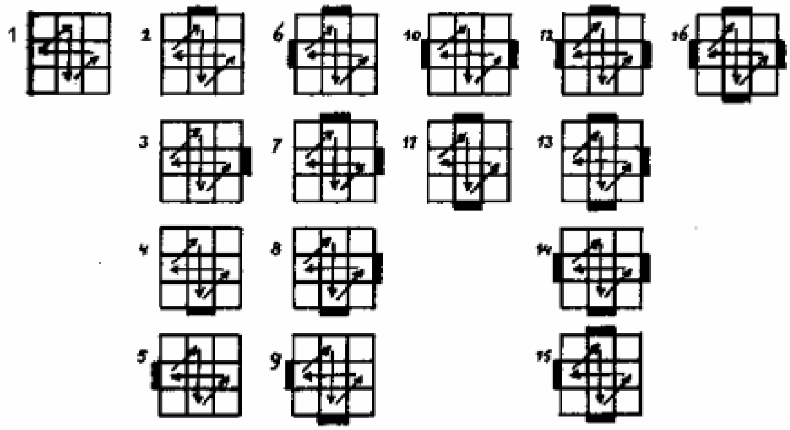
<u>Mi</u>: $U'R'MU'RUM'U'R'UR$



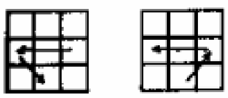
$U'R'U'RUMU'R'URM'$

<u>Mi</u>: $U'RUR'U'M'URU'R'M$

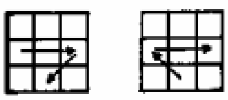
Most of the cross 4-cycles, shown below, have more than one way to solve two **redges**. To make this idea clear, we have numbered the diagrams and indicated **the** possible solutions with the use of 'partial diagrams'.



Sit. 1-6, 8, 10 and 11 are solved as follows:
Hold the U-face in such a way that two of the redges in the 4-cycle have to be moved as in one of the following partial diagrams (note that the redges at the arrow tails must not be flipped!) and execute the appropriate process.



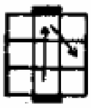
$F^2U'M'U^2MU'F^2$



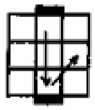
$F^2UM'U^2MUF^2$

Sit. 12-16 are solved as follows:

Hold the U-face in such a way that two of the redges in the 4-cycle have to be moved as in one of the following partial diagrams (note that both of the redges at the arrow tails are flipped!) and execute the appropriate process.



R'U'RUMU'R'URM'



RUR'U'M'URU'R'M

Sit. 7: Do not worry about the position of the U-face and turn:

$M'U^2M'U^2M'UMU^2MU^2M$

Sit. 9: Do not worry about the position of the U-face and turn:

$M^2UM^2U^2M^2UM^2$

With these processes at least two of the redges have been solved. As only U processes have been applied, at most one redge is left in the ring (Namely if it already was in the ring and the other three formed a 3-cycle). Now the remaining redge(s) can be solved, orienting the midges at the same time, using tables 3-5 from issue nr. 14.

Even though it is now always possible to solve the redges and orient the midges in two steps, it is still bad luck to come across the situations discussed in this article, as it always takes quite a long time to recognize the exact situation and act accordingly.

Having described almost everything that has been developed for this algorithm in a systematic way, we will finally point out some tricks that may speed it up a little more:

- The first (L) face.
  It is possible to solve the first face in an average of six seconds. Another article in this issue deals with this.

- R-corners and redges.
  When solving the R-corners there are two ways take already some redges into account: In the first place, there often are several alternatives for the same corner situation. If you know how these alternatives affect the redges, you may be able to choose one that puts or keeps a redge in its proper place.
  Secondly, one may insert middle layer moves in a corner process in order to affect redges differently. For instance, compare L'URU'LUR' and L'M'URU'LMUR'.

- Redges and positioning of midges.
  Very often it is possible to use the last turn of a process from tables 3-6 as the first turn of a process from table 7. This may save two turns. Also, some processes in tables 3-6 have alternatives that have a different effect on the positions of the midges. These alternatives are easy to find and they may save up to three turns.

- Combining two algorithms.
  Sometimes the last edge of the L-face would take relatively long to solve. In that case, one can leave it out and start solving the R-corners. A fast algorithm for the last L-edge, the redges and the midges exists, and will probably be discussed in a future issue.

THE ALGORITHM OF MARC WATERMAN   PART III

By Marc Waterman and Anneke Treep

This article will deal with an algorithm that many people use, though little literature is available on it. It is a complete method that resembles the algorithm described in the two previous issues. In paragraph 4 of this article both methods are combined, and it is this combination that Marc uses to solve the cube.

The method in this article is quite easy to learn and with a few extra processes it is faster than the usual layer-by-layer algorithms. With the addition of some more processes the 4x4x4 and 5x5x5 cubes can also be solved with this algorithm. This is described in paragraph 5.

In this article we often refer to issues 14 and 15, so keep them at hand. We repeat some terms and notations that were also explained in issues 14 and 15:

ring	: The slice between the R and L faces.
redge	: An edge that has its home in the R face.
midge	: An edge that has its home in the ring.
ledge	: An edge that has its home in the L face.
M	: A quarter turn of the ring, which moves UF to FD.
(R),(R)'	: (R) means one of the moves R, R^2 or R', depending on the situation. If (R)' appears in a process the inverse of the move turned at (R) earlier in the process should be applied.
hole	: An edge position in the R or L face that is not solved, i.e. not properly filled with the right edge.

A summary of the contents of this article:

1. The idea behind this algorithm.
2. The algorithm in its simplest form, easy to learn for beginners.
3. Speeding up to 20-30 seconds.
4. Combining the two algorithms.
5. Adapting the algorithm to the 4x4x4 and 5x5x5 cubes.

1. The idea behind this algorithm.

In its simplest form this algorithm solves the cube in 5 stages as follows:

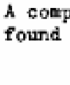
Stage 1. The first (L) face is solved except for one edge.
2. The remaining corners are solved.
3. The four redges are solved using easy three-turns processes. During this stage the hole in the L face is held at the LU position, so one may freely turn R, U and M layers.
4. The last ledge is solved.
5. The M layer is solved by first orienting the midges and then positioning them.

After the L face is solved (except for one edge of course) only four processes are necessary to solve the rest of the cube if the three-turns processes are not counted. These four are necessary to orient and position the R corners, to solve the last ledge, and to orient the midges. Solving the redges and positioning the midges can be done with three-turn processes.

2. The algorithm in its simplest form.

Stage 1. Solve the L face except for one edge. Though no special processes are needed, some useful tips are given in issue 15, page 14.

Stage 2. Hold the nearly completed L face down and solve the U corners. In theory only two processes are necessary: one to orient corners and one to position them:

 $RU'BL'BLB^2U^2R'$

 $RU^2R'U'RU^2L'UR'U'L$

A complete table that solves the U corners in one step can be found in issue 14, page 15.

Stage 3. Hold the nearly completed face again on the left side of the cube, and hold the hole in the L face at LU. The four redges are now solved one by one using three-turns processes. There are three essentially different ways in which a redge can be wrongly positioned:

a) <u>A redge is in the ring.</u>
Turn this redge to the D face with its R facelet in the D face, using a move of the ring. Turn the hole in which the redge belongs to the RU-position by a move of the R face.
If the redge is at DF turn: UM'U'
If the redge is at DB : U'MU

b) <u>A redge is in the LU hole.</u>
Turn the hole in which the redge belongs to the RU position by a move of the R face.
If the redge in the LU hole has its R facelet in the U face turn: UMU'
If the redge in the LU hole has its R facelet in the L face turn: $UM'U^2MU$

c) <u>A redge is in the R face.</u>
Turn the hole containing the redge to the RU position, and turn UMU' . This brings the redge into the ring, so go to a).

Sometimes the edge that belongs at LU is solved by accident in this stage. If not all redges have been solved yet, just ignore the solved edge at LU and let it become scrambled again.

Stage 4. So far the R and L faces have been solved with the exception of one ledge. Hold the hole in the L face where that ledge belongs at LU. The ledge may be in the ring or it may be flipped in the hole.

a) <u>The ledge is in the ring.</u>
Turn the ledge to the D face by a move of the ring, the L facelet of the ledge should be in the B or F face (only one of these possibilities is achievable by a turn of the ring).
If the ledge is at FD turn: $U'MU^2MU'$
If the ledge is at BD turn: $UM'U^2M'U$

b) <u>The ledge is flipped in its hole.</u>
Bring the ledge back to the ring with one of the processes in a); Now that the ledge is in the ring go to a).
A little faster is the following process, which solves the ledge at once: UM'UM'UM'U

Stage 5. Finally the midges in the ring are solved.
Repeating the following process and/or using conjugates of it all midges can be oriented:
UB and DB are flipped by: $UM'UM'U^2MUMU'$

To position the midges well known three-turns processes are used:
$U^2M'U^2$ or U^2MU^2
$U^2M^2U^2$
$E^2M^2E^2$

A more detailed description of this stage with more orientational processes can be found in issue 14. See also tables 6 and 7 in issue 14.

3. Speeding up to 20-30 seconds.

The method as described in paragraph 2 can be accelerated in many ways, especially stages 3, 4 and 5 can be improved upon.

Acceleration of the 2nd stage:
Using the table in issue 14, page 15, is a lot faster than orienting and positioning corners in separate stages, though the number of processes needed is of course rather large.
A good alternative is to learn for each orientation the shortest process, and all positioning processes.

Acceleration of the 3rd to 5th stage:

The redges, the ledge and the midge are now solved as follows:

a) Solve two redges using the three-turns processes from paragraph 2, stage 3.

b) Solve the third redge together with the ledge. All possible situations can be found in table 8 at the end of this article.
If the ledge happened to be solved after the two redges were completed, one can profit from this is one or both redges are in the R face. In such a case use table 3, 4 or 5 from issue 14.

c) Solving the last redge and orienting the midges is done in one process, using table 5 from issue 14. If the last redge was accidentally solved in b), then use table 6 from issue 14.

d) Position the midges, using table 7 from issue 14.

4. Combining the two algorithms.

If, after solving the corners and three edges of the first face, the centre of this face is not solved yet, then use the method from issues 14 and 15. If the centre is solved, then see below.

One has an approximate chance of 40% that it takes 4 or 5 turns to solve the last edge of the first face. In addition, it is often difficult to solve this edge fast because it is at the B or D side of the cube. In such cases it is better to just forget about this edge and to use the algortihm described in paragraph 2 of this article.

If on the other hand the edge is easy to solve it is faster to use the method from issues 14 and 15.

In short, the last-edge positions of the first face which can be solved faster with this issue are the following (hold the first face as U face):

- The 4th edge is flipped in its home position;
- The 4th edge has its U facelet in the D face;
- The 4th edge is in the E layer and cannot be solved in three moves;
- The 4th edge is not visible without moving the cube as a whole.

Sometimes it is useful to swith from the method from paragraph 2 to the method from issues 14 and 15 in a later stage: If you have chosen the method from this issue, and the last ledge happens to become solved while solving the redges, one can profit from this by switching to the appropriate stage of the method from issues 14 and 15. An example:

Say that the ledge becomes solved when the second redge is solved. Then if the 3rd and/or 4th redge are in the R face, one can use table 3 or 4 from issue 14. If both redges are in the M layer, then solve the third using process 2a or 2b from table 5 (issue 14), and orient the midges while solving the last redge, again using table 5.

5. Adapting the algorithm to the 4x4x4 and 5x5x5 cubes.

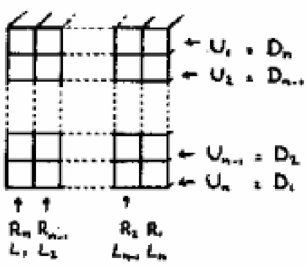
For nxnxn cubes in general, we remind you of our notation explained in issue nr. 10:

R_1 = first layer from the right

R_2 = second layer from the right

⋮

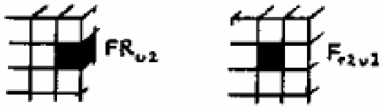
R_n = last layer from the right



This implies that $R_n = L_1'$, etc.

Some short notations are: R_{12} instead of R_1R_2 (R_{12} remains length 2)
R_{123} instead of $R_1R_2R_3$, etc.

To denote the individual pieces and positions, we use three capitals for the corners, two for the edges, and one for the centres. For the edges and centres we add subscripts that show in which layer the edge is, or in which layers the centres are, e.g.:



We will call the layers R_2, ..., R_{n-1} the M layers of the nxnxn cube.

The 4x4x4 cube can be solved in 8 stages using an adapted version of the method from paragraphs 2 and 3 from this article. Using this method Marc averages 90 seconds when solving the cube.

Stage 1. Solve the 4 centres of one colour. This is the U colour.

Stage 2. Put the four corners that have a U facelet in the proper position in respect to each other. The method from issue 15, page 15 can be used. Turn only the outer faces so that the centres can be ignored.

Stage 3. Turn the U centres between the U corners.

Stage 4. Solve the U edges except for one pair. The edges can be solved one by one or in pairs, the last being faster, though it requires some experience. The processes for this stage are simple and will not be discussed here.

Stage 5. Solve the corners of the opposite face, holding this face up. Table 1 from issue 14 can be used.

Stage 6. Apply C_F, so that the U face becomes R face, and the nearly completed first face is now L face. Hold the hole in the L face in the LU position. All redges can now be solved as in stage 4 of paragraph 2, using only R, U and M moves. Again, solving them in pairs is fastest.

Stage 7. Solving the last pair of ledges. There are five possible situations:

a) <u>The ledges are in different M layers.</u>
Put them adjacent to each other by a turn of the R_2 or L_2 layer. Then solve them using process 2a or 2b from table 5 (issue 14).

b) <u>The ledges are in the same M layer.</u>
In this case, turn the M layer containing the ledges in such a way that one ledge is in the U face, and one is in the D face. Then apply $U^2R_2^2U^2$. The ledges are now in different M layers, so go to a).

c) <u>One ledge is in the L face, and it appears to be flipped.</u>
The ledge in the L face can be in two positions, and in both cases we have two processes, because the ledge in the ring can be brought to only one of the required starting positions. First bring the ledges into the required positions, using an M move, then turn the process (remember, the first letter indicates where the L facelet should be):

ledge 1	ledge 2	
UL_{f2}	FD_{12}	$U'R'_{23}U'R'_2U'R'_{23}U'$
UL_{f2}	BD_{r2}	$UR_{23}UL'_2UR_{23}U$

ledge 1	ledge 2	
UL_{b2}	FD_{r2}	$U'R'_{23}U'L_2U'R'_{23}U'$
UL_{b2}	BD_{12}	$UR_{23}UR_2UR_{23}U$

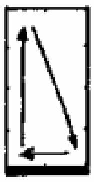
d) <u>One ledge is already solved.</u>
Again there are four processes:

ledge 1	ledge 2	
LU_{f2}	DF_{12}	$UR_2U^2L'_2U$
LU_{f2}	DB_{r2}	$U'L_2U^2R'_2U'$
LU_{b2}	DF_{r2}	$UL'_2U^2R_2U$
LU_{b2}	DB_{12}	$U'R'_2U^2L_2U'$

e) <u>The ledges are exchanged in the L face, so both appear to be flipped.</u>
Turn: $UR_{23}UR_{23}UR_{23}U$

Stage 8. Having solved all redges and ledges, we are now going to solve the midges.
First we solve the midges of the D face, which is not too difficult. It can always be done using the well known midge positioning processes for the 3x3x3, and the following processes:
$U^2L_2U^2$, $U^2L'_2U^2$, $U^2R_2U^2$, $U^2R'_2U^2$.

The only edges that still need to be solved are the midges in the U face. All possible situations are listed in table 8 at the end of this article. An example of the notation used in this table:

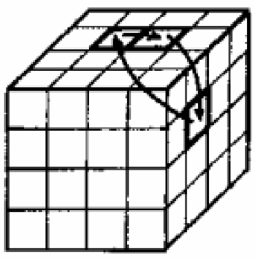


The rectangle is a representation of the midges and centres of the U face. Arrows show where the midges should go in order to get solved, and black sides show which midges appear to be flipped.
As each edge can be in only one orientation in each location, these orientational black sides could have been omitted, but we think the diagrams are clearer with them.

Stage 9. The centres can be solved using the following process, and variations and/or conjugates of it:

$L_2U_2L'_2 \cdot U' \cdot L_2U'_2L'_2\ U$

This process works on the centres as is shown in this diagram:



Apply the process slowly, so that you see how it works!

The following variations are based on the same principle:

$R'_{23}U_2R_{23} \cdot U^2 \cdot R'_{23}U'_2R_{23}\ U^2$

$R'_2U_2R_2 \cdot U^2 \cdot R'_2U'_2R_2\ U^2$

Some other useful processes are:

$B^2_2R^2_2B^2_2R^2_2$ and $R^2_2U'L^2_2UR^2_2U'L^2_2U$

The 5x5x5 and larger cubes give no new problems, and can be solved with the processes for the 4x4x4 and 3x3x3 cubes.

Some last remarks.

This was the last article in the series about the algorithm developed by Marc Waterman and Daan Krammer. The articles include nearly everything that has been developed for this algorithm since 1981. No major improvements can be made that do not violate the basic ideas of the method.

The processes on corners have been checked by Ben Jos Walbeehm's program up to ten moves; he found one acceleration. The processes in the R-U-M-group have been checked by Daan Krammer up to 13 moves.

How to read table 8.

Table 8 shows all possible situations that may arise when you want to solve one redge and the last ledge. Two redges have already been solved. There are 6 types of situations:

1 - 7 : The redge and the ledge that have to be solved are already in the LU and RU positions, but not solved yet.

8 - 15 : One of the two pieces to be solved is in RU or LU, the other is in the M layer.

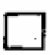
16 : Both the redge and the ledge to be solved are in the M layer.

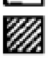
17 - 22 : The ledge is in the LU position, though not necessarily solved, and the redges are exchanged in the R face (if the redges were not exchanged, we could have found one of the situations 1 - 16).

23 - 26 : The ledge is in the R face, and forms a 3-cycle with the two redges that remain to be solved.

27 - 29 : The ledge is in the R face, and the redge that has to go to the hole occupied by the ledge is also in the R face.

To describe the situations we use squares that denote edge positions. There are four types of squares:

 : The edge position is occupied with a ledge or a redge, which is not flipped, i.e. the R or L facelet is in the R or L face.

 : The edge position is occupied with a ledge or a redge which is flipped.

 : The edge position is occupied with a redge or a ledge, but it is not important whether it is flipped or not.

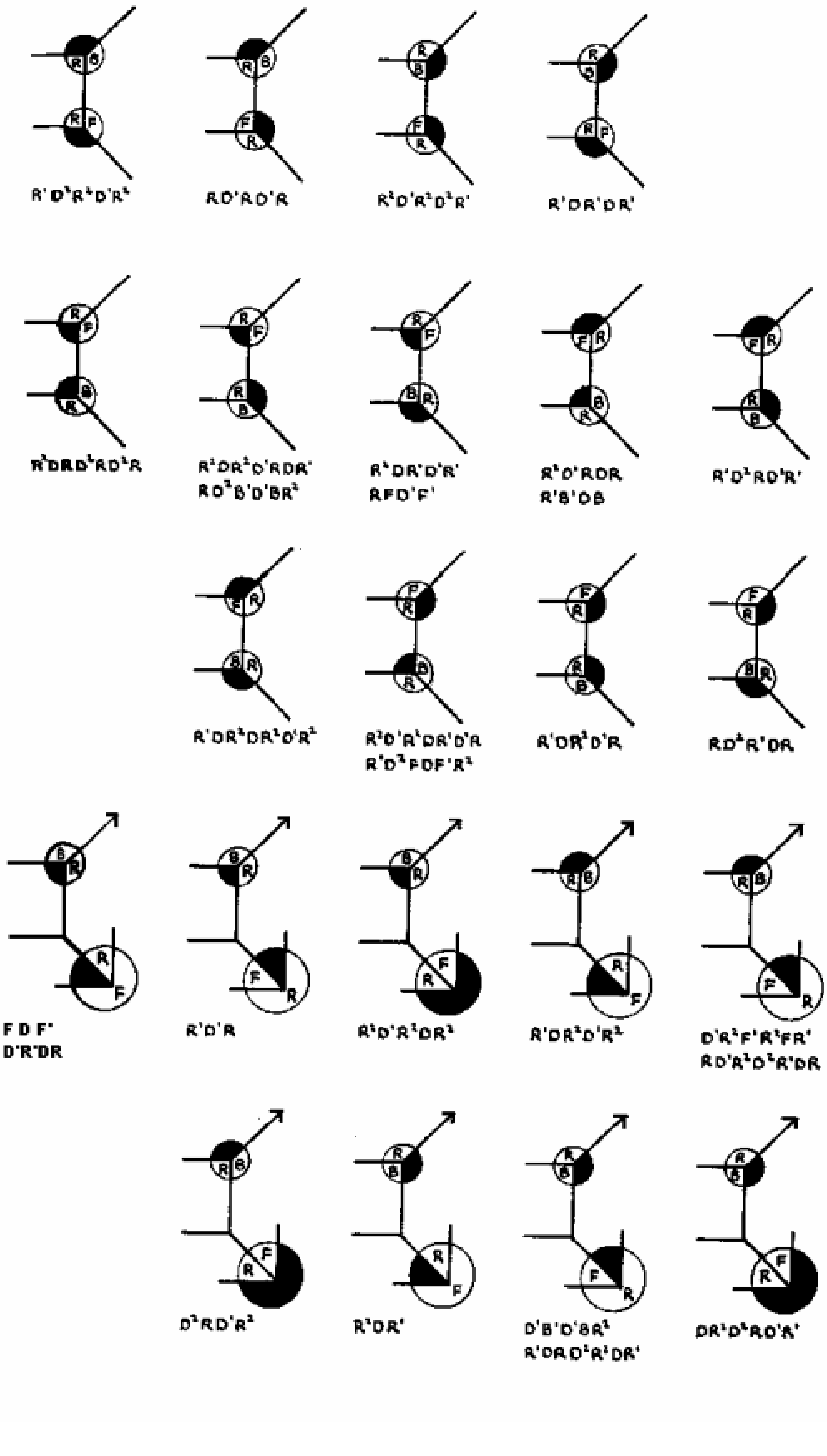
 : The edge position contains a midge.

Arrows from one edge position to another show how the piece in it has to be moved in order to be solved. Double arrows indicate an exchange of pieces.

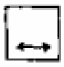
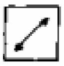
If one or both pieces to be solved are in the M layer, a pair of letters indicates where the piece(s) have to be before executing the process. This position can always be achieved by a move of the M layer. In situations 8 - 15 and 27 one piece is in the M layer, and it can be brought to only one of the two indicated positions, as they are mirror images. In situation 16 both pieces to be solved are in the M layer. The situations a-f are essentially different, but each has two processes as there are mirror images.

In short, it is always possible to produce one of the situations, using at most one R move and one M move.

Note that it not always possible to find out what a situation looks like by turning the inverse of a process!


R'D²R²D'R²
RD'RD'R
R²D'R²D²R'
R'DR'DR'
R²DRD²RD²R
R²DR²D'RDR'
RD²B'D'BR²
R²DR'D'R'
RFD'F'
R²D'RDR
R'B'DB
R'D²RD²R'
R'DR²DR²D'R²
R²D'R²DR'D'R
R'D²FDF'R²
R'DR²D'R
RD²R'DR
F D F'
D'R'DR
R'D'R
R²D'R²DR²
R'DR²D'R²
D'R²F'R²FR'
RD'R²D²R'DR
D²RD'R²
R²DR'
D'B'D'BR²
R'DRD²R²DR'
DR²D²RD'R'

1 Positioning and orienting U corners, keeping D face unimpaired

1		I	-0-
2		$B^2L^2BRB'L^2BR'B$	-9-
3		$F^2U'RU'R'UF^2URUR'$	-11-
4	B B / F F	$BU^2L^2F'L'FL'UB'$	-9-
5	R F / L F	$RUR'U'F'U'F$	-7-
6	F L / F R	$L'U'LUFUF'$	-7-
7	R L / F F	$R'URU^2R^2FRF'R$	-9-
8	F F / L R	$F'RBR'FRB'UR'$	-9-
9	B F / B F	$LUFU'F'LU^2L'U^2L^2$	-10-
10	L R / F F	$RU'BL'BLB^2U^2R'$	-9-
11	L F / F R	$L^2DL'U^2LD'L'U^2L'$	-9-
12	F R / L F	$R^2D'RU^2R'DRU^2R$	-9-
13	B B / F F	$B'R'U'RU^2FU'BUF'$	-10-
14	B F / F B	$R'UF'RBR'FRB'$	-9-
15	F F / L R	$RUBU'B'R'$	-6-

16	F R B L	$R'UF'UFU^2R^2B'R'B$	-10-
17	F R B F	$FU^2FDF'U^2FD'F^2$	-9-
18	F B L F	$L'U^2L'D'LU^2L'DL^2$	-9-
19	B F R F	$RUR'U'F'UF$	-7-
20	F L F B	$B'UBULU'L'$	-7-
21	F L F L	$L'U^2L^2F'L'FL'U^2L$	-9-
22	F B F B	$R^2U^2R'U^2R^2$	-5-
23	F R F L	$RBU^2BU^2B'U^2B^2R'$	-9-
24	L R F F	$R'FRF'U^2R^2B'R'BR'$	-10-
25	B B F F	$F^2RU^2R'_aU^2LF^2$	-8-
26	F F R L	$BU^2B^2U'B^2U'B^2U^2B$	-9-
27	F L R F	$F^2U'L'U'LU^2F^2L'U^2L$	-10-
28	R F F L	$F^2URUR'U^2F^2RU^2R'$	-10-
29	B F F B	$FR^2U^2RU^2R'U^2R'F'$	-9-
30	F F B B	$BL'BLB^2U^2RB'R'B$	-10-
31	R L F F	$F^2U'F^2RU^2R'F^2UF^2$	-9-

32	R F / L F	$LU^2L'U'LU'L'$	-7-
33	F B / B F	$FU^2F'U^2F'LFL'$	-8-
34	B F / B F	$F^2LFL^2U'L^2F'L'F^2$	-9-
35	F R / L F	$L'BLB'U^2B'U^2B$	-8-
36	R F / F L	$B'UFU'BUF'$	-7-
37	F R / F L	$R^2URU^2L'UR^2U'L$	-9-

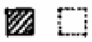
38	F L / F R	$R'U^2RUR'UR$	-7-
39	B F / F B	$F'U^2FU^2FR'F'R$	-8-
40	F B / F B	$F^2LFL^2UL^2F'L'F^2$	-9-
41	L F / F R	$RB'R'BU^2BU^2B'$	-8-
42	F L / R F	$BU'F'U\ B'U'F$	-7-
43	L F / R F	$L^2U'L'U^2RU'L^2UR'$	-9-

2 Solving two redges

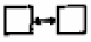
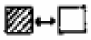
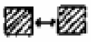
A. Both redges are in the ring. Given are the positions of the redge that belongs at RU and the other redge that will be solved.

1.	FD	BD	$U^2(R)UM^2U'(R)'U^2$	-7-
2.	UB	DF	$U^2(R)MUM^2U'(R)'U^2$	-8-
3.	UB	FD	$U^2MUM^2U(R)M'U'M'U$	-10-
4a	BD	DF	$UM^2U'(R)MUMU'$	-8-
b	FD	DB	$U'M^2U(R)M'U'M'U$	-8-
5.	FU	FD	$U^2(R)MU^2M'UM^2U(R)'U^2$	-10-

B. One redge is in the ring, the other is in the RU hole.

1a		DF	$UM'U'(R)UMU'$	-7-
b		DB	$U'MU(R)U'M'U$	-7-
2a		DF	$U^2(R)U'M'U(R)'U^2$	-7-
b		DB	$U^2(R)UMU'(R)'U^2$	-7-
3a		DF	$U^2(R)U'MU^2M^2U'(R)'U^2$	-9-
b		DB	$U^2(R)UM'U^2M^2U(R)'U^2$	-9-

3 Solving two redges already in the R-face and orienting all midges

	RU	R.	UF	UB	DB	DF		
1			O	O	O	O	$U^2(R)UM^2U^2M^2U'(R)'U^2$	-9-
2			O	X	X	O	$UM(R)UM^2U^2M'UM'(R)'U'$	-11-
3			O	X	O	X	$F^2M'F^2U(R)MUM^2U^2M'U(R)'M'U'$	-14-
4			X	X	X	X	$UM(R)UMU'M'(R)'U'M(R)U'M'U^2M'U'$	-16-
5			O	O	O	X	$UM'U'(R)U'M'U(R)'UM^2U'$	-11-
6			X	O	X	X	$UMU'(R)U'M^2U(R)'MU'MU$	-12-
7			O	O	O	O	$UMU'(R)U'M^2U(R)'UM'U'$	-11-
8			X	O	O	X	$UM(R)U'M^2UM'(R)'U'$	-9-
9			O	X	O	X	$F^2MF^2UM(R)U'M^2UM'(R)'U'$	-12-
10			X	X	X	X	$U^2(R)UMU'(R)'U^2MRU(R)U'MU(R)'U'$	-16-
11			O	O	O	O	$UM^2U^2MU(R)U'M'U^2M^2U'$	-11-
12			X	X	O	O	$U^2(R)U'MU^2M'U^2MU(R)'U^2$	-11-
13			X	O	X	O	$U^2M(R)U'MU^2M'U^2MU(R)'U^2$	-12-
14			X	X	X	X	$UM(R)UM^2U^2MUM'(R)'U'$	-11-

4 Solving two redges, of which one is in the R-face, and orienting all midges

1a		O O O	●	FD	$UM'U'(R)UMU'$
b			●	BD	$U'MU(R)U'M'U$
2a		X O O	○	DF	$U'M^2U'(R)UMU'(R)'U'M'U'$
b			○	DB	$UM^2U(R)U'M'U(R)'UMU$
3a		O X O	●	BD	$U(R)UM'U'MUM'U'(R)'M'U'$
b			●	FD	$U'(R)U'MUM'U'MU(R)'MU$
4a		O O X	●	FD	$U'(R)UMU'M'U'MU(R)'MU'$
b			●	BD	$U(R)U'M'UMUM'U'(R)'M'U$
5a		X X O	●	DF	$UM'UM'U^2M(R)UMU'$
b			●	DB	$U'MU'MU^2M'(R)U'M'U$
6a		X O X	●	DF	$UM'U'M'(R)U^2MU'MU'$
b			●	DB	$U'MUM(R)U^2M'UM'U$
7a		O X X	●	FU	$UM(R)UM'U'(R)'U^2MU$
b			●	BU	$U'M'(R)U'MU(R)'U^2M'U'$
8a		X X X	●	DF	$UM(R)UM'U'M'(R)'U'$
b			○	UF	$UM(R)UMUM'(R)'U'$
9a		O O O	●	FD	$UMU^2MU(R)M'UMU^2MU$
b			●	BD	$U'M'U^2M'U'(R)MU'M'U^2M'U'$
10a		X O O	●	FD	$UMU^2M(R)U'MU(R)'U$
b			●	BD	$U'M'U^2M'(R)UM'U'(R)'U'$
11a		O X O	●	FD	$UMU^2MU^2(R)U'MU(R)'U'$
b			●	BD	$U'M'U^2M'U^2(R)UM'U'(R)'U$
12a		O O X	○	UF	$UM'(R)U'M'UM^2(R)'U'$
b			○	UB	$U'M(R)UMU'M^2(R)'U$
13a		X X O	●	DF	$U^2(R)MU^2M'UM'U(R)'U^2$
b			●	DB	$U^2(R)M'U^2MU'MU'(R)'U^2$
14a		X O X	●	FD	$U^2M(R)UMU'(R)'U^2$
b			●	BD	$U^2M'(R)U'M'U(R)'U^2$
15a		O X X	●	DF	$U^2(R)U'M'U(R)'U^2$
b			●	DB	$U^2(R)UMU'(R)'U^2$
16a		X X X	●	FU	$U(R)U'M'UMU'M^2UM'(R)'U'$
b			●	BU	$U'(R)UMU'M'UM^2U'M(R)U$
17a		O O O	○	DF	$UM'U'(R)UM^2U^2MU$
b			○	DB	$U'MU(R)U'M^2U^2M'U'$
18a		X O O	○	FD	$UMU^2M(R)UMU^2M^2U(R)'U$
b			○	BD	$U'M'U^2M'(R)'U'M'U^2M^2U'(R)'U'$
19a		O X O	○	FD	$UMU^2MU^2(R)U'M'U^2M^2U'(R)'U'$
b			○	BD	$U'M'U^2M'U^2(R)UMU^2M^2U(R)'U$
20a		O O X	●	UF	$UM'(R)U'M'UM'(R)'U^2MU$
b			●	UB	$U'M(R)UMU'M(R)'U^2M'U'$
21a		X X O	●	DF	$U^2(R)MU^2M'UMU^2M^2U'(R)'U^2$
b			●	DB	$U^2(R)M'U^2MU'M'U^2M^2U(R)'U^2$
22a		X O X	●	FD	$U^2M(R)UM'U^2M^2U(R)'U^2$
b			●	BD	$U^2M'(R)U'MU^2M^2U'(R)'U^2$
23a		O X X	●	DF	$U^2(R)U'MU^2M^2U'(R)'U^2$
b			●	DB	$U^2(R)UM'U^2M^2U(R)'U^2$
24a		X X X	●	FD	$UMU^2(R)U'MUM(R)'U$
b			●	BD	$U'M'U^2(R)UM'U'M'(R)'U'$

5 Solving one redge, and orienting all midges.

A. Redge in R-face.

	UF	UB	DB	DF	hole		
1.	O	X	O	O	not RU	U(R)U'M'U^{2}M^{2}U'(R)'U'	-9-
2.	X	O	X	X	RU	U'M'U'M'U'M'U'	-7-

B. Redge in the ring.

				hole			
1.a	O	O	O	RU	FD	UMU^{2}MU	-5-
b					BD	U'M'U^{2}M'U'	-5-
2.a	X	O	O	not RU	UF	U(R)U'M'U(R)'U'	-7-
b					UB	U'(R)UMU'(R)'U	-7-
3a	O	X	O	not RU	DF	U(R)UM'U'(R)'U'	-7-
b					DB	U'(R)U'MU(R)'U	-7-
4.a	O	O	X	not RU	DF	U'(R)UM'U'(R)'U	-7-
b					DB	U(R)U'MU(R)'U'	-7-
5.a	X	X	O	RU	DF	UMU'M'UM'U'	-7-
b					DB	U'M'UMU'MU	-7-
6a	X	O	X	RU	DF	UMUM'U'M'U'	-7-
b					DB	U'M'U'MUMU	-7-
7a	O	X	X	RU	FU	U^{2}M'UM'UMU'MU	-9-
b					BU	U^{2}MU'MU'M'UM'U'	-9-
8a	X	X	X	RB	UF	UR'FU'M'UF'RU'	-9-
b				RF	UB	U'RB'UMU'BR'U	-9-

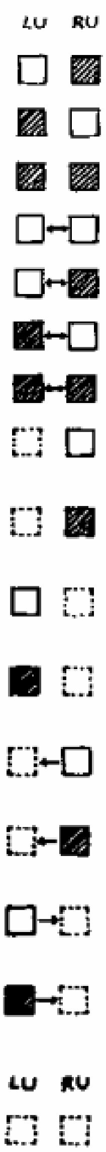
6 Orienting midges.

	UF UB DB DF		
1.	O X X O	$U^2FMF'U^2FM'F'$	-8-
2.	X O X O	$FMF'U^2FM'F'U^2$	-8-
3	X X X X	$UR'MUMU'M^2RU'R'UMU'$	-13-

7 Positioning all midges

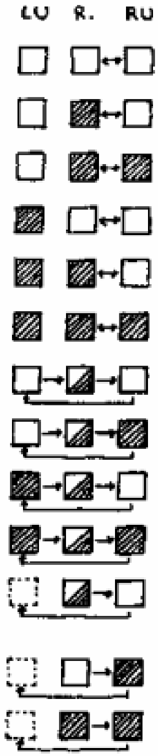
1.	(UB, UF) (DB, DF)	$U^2M^2U^2$
2.	(UB, DF) (UF, DB)	E^2ME^2
3.	(UB, UF, DF)	F^2MF^2

Table 8. Solving 1 redge and the last ledge.



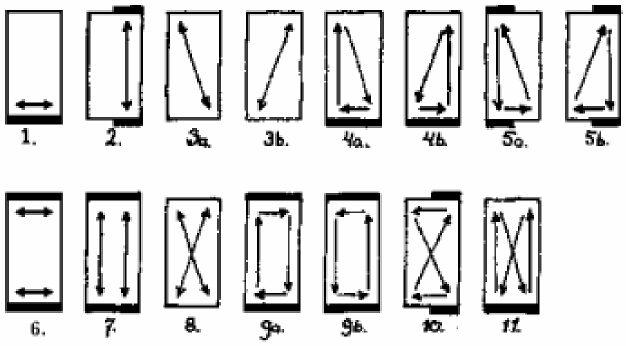
	LU / RU	
1.		$U'M'U'M'U'M'U'$
2.		$UM'UM'UM'U$
3.		$U'M'U^2M\ U^2M'U$
4.		$U'M^2U^2M^2U'$
5.		$UMU'MU'M'U^2M^2U'$
6.		$UM'U'M'U'MU^2M^2U'$
7.		$U'M'U^2M'U^2M'U$
8.	FD	$U'MU^2MU'$
	BD	$UM'U^2M'U$
9.	DF	UMU^2M^2U
	DB	$U'M'U^2M^2U'$
10.	FD	UMU^2MU
	BD	$U'M'U^2M'U'$
11.	DF	$U'MU^2M^2U'$
	DB	$UM'U^2M^2U$
12.	FD	$U'M'U^2MU'$
	BD	$UMU^2M'U$
13.	DF	$UM'U'$
	DB	$U'MU$
14.	FD	$UM'U^2MU$
	BD	$U'MU^2M'U'$
15.	DF	$U'M'U$
	DB	UMU'

	ledge	redge	
16a.	FD	BD	UM^2U'
	BD	FD	$U'M^2U$
b.	FD	DB	$M'U'MU'M'UM^2U$
	BD	DF	$MUM'UMU'M^2U'$
c.	DF	BD	$MUM'UMUM^2U$
	DB	FD	$M'U'MU'M'U'M^2U'$
d.	DF	DB	$U'M'U^2MU^2MU$
	DB	DF	$UMU^2M'U^2M'U'$
e.	UF	DB	$U^2M'UM^2U$
	UB	DF	$U^2MU'M^2U'$
f.	UF	BD	$U'M'UMUM^2U'$
	UB	FD	$UMU'M'U'M^2U$



	LU R. RU	
17.		$U'M^2U(R)U'M^2U$
18.		$U'M^2U(R)U'M^2U$
19.		$U^2(R)UMU'(R)'U^2$
20.		$U'MU(R)MUMU'$
21.		$U'MU(R)MUMU'$
22.		$U'MU(R)UM'U^2MU$
23.		$U'M^2U(R)UM^2U'$
24.		$U'MU'(R)UM'U'(R)'U^2$
25.		$UM'U'(R)M'U'MU^2M^2U$
26.		$(R)U'M'U(R)'U'MU$
27.	FD	$U'M'U(R)UMU'$
	BD	$UMU'(R)U'M'U$
28.		$(R)U'M'U(R)'U'MU$
29.		$(R)U'M^2U(R)'UM'U'$

Table 9. Solving the midges of the U face.



1. $L_2^2 B^2 D^2 R_2' D' F_2 D' B^2 D F_2' D' L_2^2$

2. $R_2' D' F_2 D' B^2 D F_2' D' B^2 D^2$

3a. $C_L \cdot U R_{12}' U R_2' U' R R_2^2 U' R' U R_2' U' R$

b. $C_L \cdot U R_{123}' U L_2 U' R_{12} L_2^2 U' R' U L_2 U' R$

4a. $L_2' U^2 B D_2 B' U^2 B D_2' B' L_2$

b. $R_2 U^2 B' D_2' B U^2 B' D_2 B R_2'$

5a. $L_2' B D_2 B' U^2 B D_2' B' U^2 L_2$

b. $R_2 B' D_2' B U^2 B' D_2 B U^2 R_2'$

6. $C_R \cdot U R_{23} U R_{23} U^2 R_{23}' U R_{23}' U R_{23}' U^2 R_{23}$

7. $C_R \cdot U R_{23} U R_{23} U^2 R_{23}' U R_{23}' U' R_2^2 U^2 R_2^2 U^2 R_2^2$

8. $R_2^2 U^2 R_2^2 U^2 R_2^2$

9a. $R_2^2 U^2 R_2' U^2 R_2^2$

b. $R_2^2 U^2 R_2 U^2 R_2^2$

10. $F R' D R_2^2 B^2 R_2' B^2 R_2^2 D' R F'$

11. $L_2 U^2 F' D_2' F U^2 F' D_2 F L_2 U^2 L_2 U^2 L_2^2$